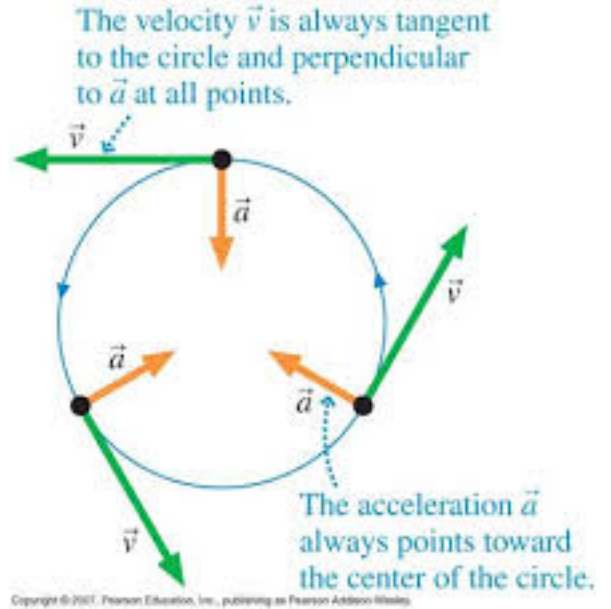


# Uniform Circular Motion

**Linear or Tangential Velocity** =  $\frac{2\pi r}{\tau}$

Uniform circular motion can be described as the motion of an object in a circle at a constant speed. As an object moves in a circle, it is constantly changing its direction. At all instances, the object is moving tangent to the circle. Since the direction of the **velocity** vector is the same as the direction of the object's motion, the velocity vector is directed tangent to the circle as well.



**Comprehension Questions:**

1. What remains constant?
2. What is constantly changing?
3. Since the \_\_\_ vector is changing (direction), we can say the object is \_\_\_.

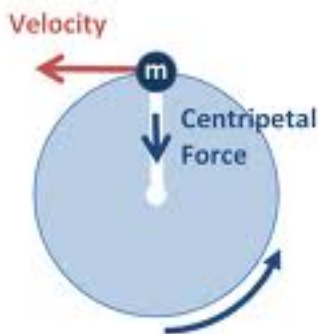
**Centripetal Acceleration** ( $a_c$ ) =  $\frac{v^2}{r}$

An object moving in a circle is **accelerating**. Accelerating objects are objects which are changing their velocity - either the speed (i.e., magnitude of the velocity vector) or the direction. An object undergoing uniform circular motion is moving with a constant speed. Nonetheless, it is accelerating due to its change in direction. The direction of the acceleration is inwards.

**Comprehension Questions:**

4. What remains constant?
5. Why is it accelerating, if the speed is not changing?
6. In what direction will in object moving in a circular path, always accelerate?

$F_{net} = \text{Force}_{centripetal} = F_c = \frac{mv^2}{r}$



The final motion characteristic for an object undergoing uniform circular motion is the **net force**. The net force acting upon such an object is directed towards the center of the circle. The net force is said to be an inward or *centripetal* force. Without such an inward force, an object would continue in a straight line, never deviating from its direction. Yet, with the inward net force directed perpendicular to the velocity vector, the object is always changing its direction and undergoing an inward acceleration.

**Comprehension Questions:**

7. If an object is accelerating it must have a \_\_\_ acting on it.
8. If the mass of the revolving object were to increase, in order to make it move with the same speed and radius the \_\_\_ must also increase.
9. If the mass and speed remain constant, what happens to the force when radius decreases?

## Frequency (f) and Period (T)

For objects moving in circular paths, we can characterize their motion around the circle using the terms **frequency (f) and period (T)**. The frequency of an object is the number of revolutions the object makes in a complete second. It is measured in units of Hertz (Hz). In similar fashion, the period of an object is the time it takes to make one complete revolution. Since the period is a time interval, it is measured in units of seconds. We can relate period and frequency using the equations:

$$f = \frac{1}{T} \quad T = \frac{1}{f} = \frac{\# \text{ seconds}}{\# \text{ revolutions in that many seconds}}$$

### Comprehension Questions:

A 500g toy train completes 10 laps of its circular track in 100s.

10. What is the period?

11. What is the frequency?

12. What does “centripetal” mean? \_\_\_\_\_

13. Fill in the chart with the missing information

Key Terms	Meaning	Symbol	Formula
Centripetal acceleration		$a_c$	
Centripetal force		$F_c$	
Frequency		$f$	
Linear Speed		$v$	
Period		$T$	

### Practice Problems

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14. A 500g toy train completes 10 laps of its circular track in 1 min 40s. If the diameter of the track is 1m, find the train's linear speed ( $v$ ), centripetal acceleration ( $a_c$ ), centripetal force ( $F_c$ ), period ( $T$ ), and frequency ( $f$ ).

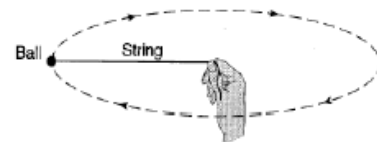
15. A 0.2 kg object is twirled in a horizontal plane on a 0.5 m rope at 1 revolution every 1.5 seconds. From this it can be found that the mass has a tangential speed of 2.09 m/s.

(a) What is the centripetal acceleration of the object?

(b) What is the centripetal force experienced by the object? Where is the force directed?

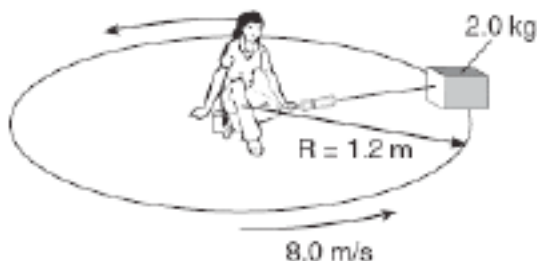
(c) Describe how the acceleration and velocity vectors are related in uniform circular motion.

16. The diagram shows a student spinning a 0.10-kilogram ball at the end of a 0.50-meter string in a horizontal circle at a constant speed of 10. m/s. [Neglect air resistance.] If the magnitude of the force applied to the string by the student's hand is increased, the magnitude of the acceleration of the ball in its circular path will



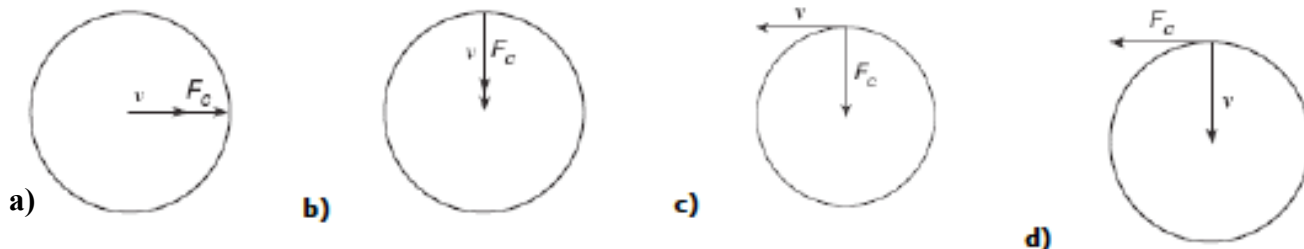
- a) decrease b) can't be determined c) increase d) remain the same

17. The diagram shows a student seated on a rotating circular platform, holding a 2.0-kilogram block with a spring scale. The block is 1.2 meters from the center of the platform. The block has a constant speed of 8.0 meters per second. [Frictional forces on the block are negligible.] Which statement best describes the block's movement as the platform rotates?



- a) Its velocity is directed tangent to the circular path, with an inward acceleration.  
 b) Its velocity is directed perpendicular to the circular path, with an outward acceleration.  
 c) Its velocity is directed perpendicular to the circular path, with an inward acceleration.  
 d) Its velocity is directed tangent to the circular path, with an outward acceleration.

18. A  $1.0 \times 10^3$  kg car travels at a constant speed of 20. m/s around a horizontal circular track. Which diagram correctly represents the direction of the car's velocity ( $v$ ) and the direction of the centripetal force ( $F_c$ ) acting on the car at one particular moment?



19. A child sitting 1.10m from the center of a merry-g-round moves with a speed of 1.25 m/s. Calculate  
 a) the centripetal acceleration of the child  
 b) the net force exerted on the child (mass = 25.0kg)

20. Which factor, when doubled, would produce the greatest change in the centripetal force acting on the car?

- a) velocity of the car                      b) mass of the car  
 c) radius of the track                      d) weight of the car

21. A ball of mass  $M$  at the end of a string is swung in a horizontal circular path of radius  $R$  at constant speed  $V$ . Which combination of changes would require the greatest increase in the centripetal force acting on the ball?

- a) halving  $V$  and halving  $R$                       c) doubling  $V$  and doubling  $R$   
 b) halving  $V$  and doubling  $R$                       d) doubling  $V$  and halving  $R$

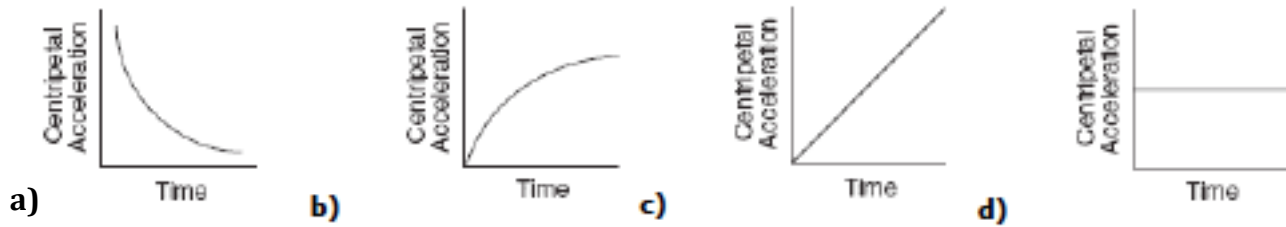
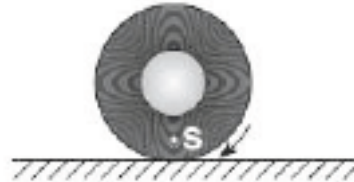
22. Centripetal force ( $F_c$ ) acts on a car going around a curve. If the speed of the car were twice as great, the magnitude of the centripetal force necessary to keep the car moving in the same path would be

- a)  $F_c$    b)  $F_c/2$    c)  $2F_c$    d)  $4F_c$

23. A child is riding on a merry-go-round. As the speed of the merry-go-round is doubled, the magnitude of the centripetal force acting on the child

- a) is doubled   b) remains the same   c) is quadrupled   d) is halved

24. In the diagram below, S is a point on a car tire rotating at a constant rate. Which graph best represents the magnitude of the centripetal acceleration of point S as a function of time?



25. The maximum force a road can exert on the tires of a 1600kg car is 8400N. What is the maximum velocity at which the car can round a turn of radius 130m?

**Answers:**  
 1) speed 2) direction 3) velocity, accelerating 4) speed 5) changing direction 6) toward the center 7) net force 8) net force 9) increases 10) 10s 11) .1Hz 12) center-seeking 13)  $a_c = \frac{v^2}{r}$  = the acceleration of an object moving in a circular path,  $F_{\text{FORCE centripetal}} = F_c = \frac{mv^2}{r}$  = the center directed net force of an object moving in a circular path,  $f$  = the number of rotations per second,  $v = \frac{r}{2\pi T}$  = the distance it travels (radius) per unit of time (s)  $T$  = the time to complete on rotation 14)  $0.3 \text{ m/s}$ ,  $0.197 \text{ m/s}^2$ ,  $0.099 \text{ N}$ ,  $10\text{s}$ ,  $0.1\text{Hz}$  15)  $8.7 \text{ m/s}^2$ ,  $1.7\text{N}$ , toward the center, if net force or  $F_c$  is directed toward the center the object accelerates toward the center. Velocity is the speed at which it maintains as it is moving around the circle. 16) c   17) a   18) c   19) a =  $1.42 \text{ m/s}^2$    b =  $35.5\text{N}$    20) a   21) d   22) d   23) c   24) d   25) 26 m/s